## 973. Maximum sum on a tree

Given a tree with *n* vertices, where the vertex numbered *i* (1 ≤ *i* ≤ *n*) contains *ci* coins. Select a subset of vertices such that no two of them are adjacent (i.e., connected by an edge), and the sum of coins in the selected vertices is maximized.



**Input.** The first line contains the number of vertices *n* (1 ≤ *n* ≤ 105) in a tree. Each of the next *n* – 1 lines contains  two integers *u* and *v* (1 ≤ *u*, *v* ≤ *n*), defining an edge in the tree. The last line contains *n* non-negative integers *c*1, ... *cn* – the number of coins in each vertex of the tree.

**Output.** Print the maximum possible sum of coins that can be obtained by selecting a subset of vertices in the tree with no adjacent vertices.

|  |  |
| --- | --- |
| **Sample input 1** | **Sample output 1** |
| 5  1 2  1 3  2 4  2 5  1 5 7 1 2 | 12 |
|  |  |
| **Sample input 2** | **Sample output 2** |
| 5  1 2  1 3  2 4  2 5  3 7 5 10 1 | 16 |

## SOLUTION

**dynamic programmng - trees**

# Algorithm analysis

Let *v* be a vertex of the tree. Let’s define:

* dp1(*v*) as the maximum sum of coins that can be collected from the subtree rooted at *v* if we take the coins in vertex *v*.
* dp2(*v*) as the maximum sum of coins that can be collected from the subtree rooted at *v* if we do not take the coins in vertex *v*.

Then the answer to the problem will be max(dp1(1), dp2(1)), assuming the first vertex is taken as the root of the tree.

Let’s define the given functions recursively:

* If we take the coins in vertex *v*, then we cannot take coins from its children:

dp1(*v*) = *cv* + , where *to*1, …, *tok* are the sons of vertex *v*.

* If we do not take the coins in vertex *v*, then we can choose either to take or not take coins from its children. We select the option with the maximum sum of coins:

dp2(*v*) = , where *to*1, …, *tok* are the sons of vertex *v*.

If *v* is a leaf with *cv* coins, then the functions take the following values: dp1(*v*) = *cv* and dp2(*v*) = 0.

**Example**

Let’s assign the labels dp1(*v*) / dp2(*v*) to the vertices of the trees from the examples.



For the first example, we have:

* dp1(1) = *c*1 + dp2(2) + dp2(3) = 1 + 3 + 0 = 4;
* dp2(1) = max(dp1(2), dp2(2)) + max(dp1(3), dp2(3)) = 5 + 7 = 12;
* dp1(2) = *c*2 + dp2(4) + dp2(5) = 5 + 0 + 0 = 5;
* dp2(2) = max(dp1(4), dp2(4)) + max(dp1(5), dp2(5)) = 1 + 2 = 3;

For the second example, we have:

* dp1(1) = *c*1 + dp2(2) + dp2(3) = 3 + 11 + 0 = 14;
* dp2(1) = max(dp1(2), dp2(2)) + max(dp1(3), dp2(3)) = 11 + 5 = 16;
* dp1(2) = *c*2 + dp2(4) + dp2(5) = 7 + 0 + 0 = 7;
* dp2(2) = max(dp1(4), dp2(4)) + max(dp1(5), dp2(5)) = 10 + 1 = 11;

**Exercise**

Assign the labels dp1(*v*) / dp2(*v*) to the vertices of the tree:



# Algorithm implementation

Declare the arrays.

vector<vector<int> > g;

vector<int> dp1, dp2, cost;

The ***dfs*** function implements depth-first search. Compute the values of dp1 and dp2 at all vertices of the tree.

void dfs(int v, int p = -1)

{

dp1[v] = cost[v];

dp2[v] = 0;

for (int to : g[v])

{

if (to == p) continue;

dfs(to, v);

dp1[v] += dp2[to];

dp2[v] += max(dp1[to], dp2[to]);

}

}

The main part of the program. Read the tree and the array of coins.

scanf("%d",&n);

g.resize(n+1);

for(i = 1; i < n; i++)

{

scanf("%d %d",&u,&v);

g[u].push\_back(v);

g[v].push\_back(u);

}

dp1.resize(n+1); dp2.resize(n+1);

cost.resize(n+1);

for(i = 1; i <= n; i++)

scanf("%d",&cost[i]);

Let the root of the tree be at vertex 1. Start the depth-first search from there.

dfs(1);

Compute and print the answer.

ans = max(dp1[1], dp2[1]);

printf("%d\n",ans);