10203. Arctic network

The Ministry of National Defense (MND) plans to connect several northern outposts into a single wireless network. Two different communication technologies must be used to build the network: each outpost will be equipped with a radio transceiver, and some of them will additionally have a satellite channel.

Any two outposts equipped with a satellite channel can communicate with each other via satellite, regardless of the distance between them. Otherwise, two outposts can communicate by radio only if the distance between them does not exceed *d*, which depends on the power of the transceivers. The higher the power, the larger the value of *d*, but the cost of the equipment also increases. For economic and maintenance reasons, all transceivers must be identical, meaning that the value of *d* must be the same for all pairs of outposts.

You must determine the minimum value of  required so that communication is possible between any two outposts – either directly or indirectly (through other outposts).

**Input.** The first line contains the number of test cases *n*. The first line of each test case contains the number of satellite channels *s* (1 ≤ *s* ≤ 100) and the number of outposts *p* (*s* < *p* ≤ 500). Each of the next *p* lines contains the coordinates (*x*, *y*) of an outpost in kilometers (integers from 0 to 10000).

**Output.** For each test case, print the minimum value of *d* required to connect all outposts into a single network. Print the result with exactly two digits after the decimal point.

|  |  |
| --- | --- |
| **Sample input** | **Sample output** |
| 1  2 4  1 0  3 0  6 0  7 2 | 2.24 |

## SOLUTION

**graphs – Prim algorithm**

# Algorithm analysis

Start Prim’s algorithm. Construct an array *dist*, where dist[*i*] stores the length of the shortest edge from MST that runs to the vertex *i*. That is, MST consists of these edges. Moreover, dist[0] = 0, since we start the algorithm from the vertex 0.

At the end of Prim’s algorithm, sort the *dist* array. The most distant outposts should be connected by satellite channels. They should be placed in those *s* outposts that connect the longest edges from *dist* (*s* – 1 edges connect *s* outposts). Therefore, the desired value of *d* will be the length of the edge that is the *s*-th from the end of *dist*.

**Example**

Consider the graph given in a sample.



Put two satellite channels at points (3; 0) and (6; 0). Thus, the outposts in these coordinates are connected via satellite. Find the largest one among the remaining edges of MST. This will be the edge connecting the points (6; 0) and (7; 2), its length is 2.24.

# Algorithm implementation

Declare global arrays. Store the coordinates of the outposts in (x[*i*], y[*i*]). used[*i*] = 1, if vertex *i* is included in MST.

#define MAX 501

#define INF 0x3F3F3F3F

int x[MAX], y[MAX];

int used[MAX], dist[MAX];

The function ***dist2*** computes the squared distance between outposts *i* and *j*.

int dist2(int i, int j)

{

return (x[j] - x[i])\*(x[j] - x[i]) + (y[j] - y[i])\*(y[j] - y[i]);

}

Function ***Prim*** implements the Prim’s algorithm.

void Prim(void)

{

Initialize the arrays.

memset(dist, 0x3F, sizeof(dist));

memset(used, 0, sizeof(used));

Start building the MST from the vertex 0. Initialize dist[0] = 0, used[0] = 1.

int i, j, cur = 0;

dist[cur] = 0;

used[cur] = 1;

Make *n* – 1 iterations. At each iteration, add one vertex to MST (so *n* – 1 vertices should be added to MST).

for (i = 1; i < n; i++)

{

The current vertex is *cur*. Iterate over the edges (*cur*, *j*) and recalculate the value of dist[*j*]. Thus dist[*j*] stores the current shortest distance from vertex *j* to the current MST.

for (j = 0; j < n; j++)

if (!used[j] && (dist2(cur, j) < dist[j]))

dist[j] = dist2(cur, j);

We are looking for the shortest edge that will be included in MST. To do this, look for the smallest dist[*j*] such that the vertex *j* does not yet belong to MST (used[*j*] = 0). The number of the vertex with the smallest dist[*j*] is stored into *cur* (it becomes the current one).

int min = INF;

for (j = 0; j < n; j++)

if (!used[j] && (dist[j] < min))

{

min = dist[j];

cur = j;

}

Vertex *cur* is included to MST.

used[cur] = 1;

}

}

The main part of the program. Process *tests* tests.

scanf("%d", &tests);

while (tests--)

{

Read the input data.

scanf("%d %d", &s, &n);

for (i = 0; i < n; i++)

scanf("%d %d", &x[i], &y[i]);

Run the Prim’s algorithm.

Prim();

Sort the array dist.

sort(dist, dist + n);

Print the answer.

printf("%.2lf\n", sqrt(dist[n - s]));

}

**Java realization**

**import** java.util.\*;

**public** **class** Main

{

**static** **int** *n*;

**static** **int** *x*[], *y*[];

**static** **int** *used*[], *dist*[];

**static** **int** dist2(**int** i, **int** j)

{

**return** (*x*[j] - *x*[i])\*(*x*[j] - *x*[i]) + (*y*[j] - *y*[i])\*(*y*[j] - *y*[i]);

}

**static** **void** Prim()

{

*dist* = **new** **int**[*n*];

Arrays.*fill*(*dist*, Integer.***MAX\_VALUE***);

*used* = **new** **int**[*n*];

**int** i, j, cur = 0;

*dist*[cur] = 0;

*used*[cur] = 1;

**for** (i = 1; i < *n*; i++)

{

**for** (j = 0; j < *n*; j++)

**if** (*used*[j] == 0 && *dist2*(cur, j) < *dist*[j])

*dist*[j] = *dist2*(cur, j);

**int** min = Integer.***MAX\_VALUE***;

**for** (j = 0; j < *n*; j++)

**if** (*used*[j] == 0 && *dist*[j] < min)

{

min = *dist*[j];

cur = j;

}

*used*[cur] = 1;

}

}

**public** **static** **void** main(String[] args)

{

Scanner con = **new** Scanner(System.***in***);

**int** tests = con.nextInt();

**while** (tests-- > 0)

{

**int** s = con.nextInt();

*n* = con.nextInt();

*x* = **new** **int**[*n*];

*y* = **new** **int**[*n*];

**for** (**int** i = 0; i < *n*; i++)

{

*x*[i] = con.nextInt();

*y*[i] = con.nextInt();

}

*Prim*();

Arrays.*sort*(*dist*);

System.***out***.println(Math.*sqrt*(*dist*[*n* - s]));

}

con.close();

}

}