11317. K-variance

Consider the variance of the sequence *a*1, *a*2, ..., *an* as



where



Consider the K-variance as the variance of the consecutive subsequence of length *k*.

Your task is to calculate all (*n* – *k* + 1) K-variances for the given sequence and *k*.

Formally, the *i*-th (1 ≤ *i* ≤ *n* – *k* + 1) K-variance *ri* is the variance of sequence {*ai*, *ai*+1, ..., *ai*+*k*-1​}.

**Input.** The first line contains two integers *n* and *k* (2 ≤ *k* ≤ *n* ≤ 105).

The second line contains *n* integers *a*1, *a*2, ..., *an* (|*ai*| ≤ 100).

**Output.** Print (*n* – *k* + 1) lines with real numbers *r*1, *r*2, ..., *rn*-*k*+1​.

The answer is considered correct if its absolute or relative error does not exceed 10-4.

|  |  |
| --- | --- |
| **Sample input 1** | **Sample output 1** |
| 3 2  1 3 2 | 1.41421356  0.70710678 |
|  |  |
| **Sample input 2** | **Sample output 2** |
| 5 3  1 3 2 4 5 | 1.00000000  1.00000000  1.52752523 |

## SOLUTION

**mathematics**

**Algorithm analysis**

Let’s consider the sum:

 =  =  =

 –  +  =

 –  +  =  – 

Now, the formula for variance can be rewritten as:

 = 

If we maintain the sum of the elements of the sequence {*ai*, *ai*+1, ..., *ai*+*k*-1​} (i.e., *ai* + *ai*+1 + ... + *ai*+*k*-1) and the sum of their squares (), then it is possible to calculate the desired variance for the segment in constant time.

**Example**

Let’s expand the numerator of the fraction in the variance for *n* = 3:

 +  +  =

=  –  –  –  +

+  + +  =

=  –  +  =

=  –  +  =

=  – 

**Algorithm implementation**

Read the input data.

scanf("%d %d", &n, &k);

a.resize(n);

for (i = 0; i < n; i++)

scanf("%d", &a[i]);

On the segment [*i* – *k* + 1; *i*] let’s maintain two variables:

* The sum *sum* = *ai*-*k*+1 + … + *ai*-1 + *ai*,
* The sum of squares *sum*2 = 

sum = sum2 = 0;

Iterate through the elements of the sequence.

for (i = 0; i < n; i++)

{

Update the values of *sum* and *sum*2.

sum += a[i];

sum2 += a[i] \* a[i];

If there is a window [*i* – *k* + 1; *i*] of length *k*, print the result for it.

if (i >= k - 1)

{

printf("%lf\n", sqrt((sum2 - (sum \* sum) / k) / (k - 1)));

Remove the element with index *i* – *k* + 1 from the considered sequence.

sum -= a[i - k + 1];

sum2 -= a[i - k + 1] \* a[i - k + 1];

}

}