## 2269. Connected components

An undirected unweighted graph is given. Find the number of its connected components.

**Input.** The first line contains the number of vertices *n* (*n* ≤ 100). Then follow *n* lines with *n* numbers each – the adjacency matrix of the graph. In the *i*-th row, at the *j*-th position there is 1 if vertices *i* and *j* are connected by an edge, and 0 otherwise. The main diagonal of the matrix contains zeros. The matrix is symmetric with respect to the main diagonal.

**Output.** Print one integer – the number of connected components of the given graph.

|  |  |
| --- | --- |
| **Sample input** | **Sample output** |
| 6  0 1 1 0 0 0  1 0 1 0 0 0  1 1 0 0 0 0  0 0 0 0 1 0  0 0 0 1 0 0  0 0 0 0 0 0 | 3 |

## SOLUTION

**graphs – connected components**

# Algorithm analysis

To find the number of connected components in a graph, a disjoint-set data structure (Union-Find) can be used.

Initially, each vertex is placed in its own set, and each vertex is the representative of that set. Then, for each edge (*u*, *v*), the sets containing vertices *u* and *v* are united. After processing all edges, the number of connected components equals the number of sets in the structure.

This problem can also be solved using depth-first search. In this case, the number of times the ***dfs*** procedure is called will equal the number of connected components in the graph.

**Example**

The graph given in the example looks as follows:



Initially, each vertex is placed in its own set, where it acts as the representative.



Then, for each edge (*u*, *v*), the sets containing vertices *u* and *v* are united. After processing all edges, two vertices will belong to the same connected component if they have the same representative.









The number of connected components is equal to the number of sets in the disjoint-set structure. This number is determined by the representatives – those vertices *v* for which the condition parent[*v*] = *v* holds.

In the given example, the representatives are vertices 3, 5, and 6. Therefore, the graph contains three connected components.

# Algorithm implementation

MAX stores the maximum number of vertices in the graph. In the array mas[*i*], the number of the vertex to which vertex *i* points is stored.

#define MAX 102

int mas[MAX];

The function ***Repr*** returns the representative vertex of the set to which vertex *n* belongs. To do this, we follow the pointers sequentially until we reach the representative of the set (a vertex whose pointer points to itself).

int Repr(int n)

{

while (n != mas[n]) n = mas[n];

return n;

}

The ***Union*** function merges two sets containing vertices *x* and *y*. First, their representatives are found – let them be *x*1 and *y*1. If *x*1 = *y*1, then the vertices already belong to the same set, and no further action is required. Otherwise, the pointer of the representative *x*1 is redirected to *y*1.

void Union(int x,int y)

{

int x1 = Repr(x),y1 = Repr(y);

if (x1 == y1) return;

mas[x1] = y1;

}

The main part of the program. Each vertex initially points to itself (mas[*i*] = *i*).

scanf("%d",&n);

for (i = 1; i <= n; i++) mas[i] = i;

Read the adjacency matrix. For each edge (*i*, *j*) where *i* < *j*, perform a union of the sets containing vertices *i* and *j*.

for (i = 1; i <= n; i++)

for (j = 1; j <= n; j++)

{

scanf("%d",&value);

if (i > j) continue;

if (value) Union(i,j);

}

The variable *count* stores the number of connected components. This number is equal to the number of representative vertices of the sets – that is, those vertices that point to themselves.

count = 0;

for (i = 1; i <= n; i++)

if (mas[i] == i) count++;

Print the answer.

printf("%d\n",count);

**Algorithm implementation – depth first search**

Declare the arrays.

#define MAX 102

int g[MAX][MAX], used[MAX];

The ***dfs*** function performs a depth-first search of the graph, starting from vertex *v*.

void dfs(int v)

{

used[v] = 1;

for (int i = 0; i < n; i++)

if (g[v][i] && !used[i]) dfs(i);

}

The main part of the program. Read the input data.

scanf("%d",&n);

for(i = 0; i < n; i++)

for(j = 0; j < n; j++)

scanf("%d",&g[i][j]);

The variable *res* stores the number of connected components.

res = 0;

A depth-first search is performed on the disconnected graph. Each call to the ***dfs*** function starts from an unvisited vertex, so the number of ***dfs*** calls equals the number of connected components in the graph.

memset(used,0,sizeof(used));

for (i = 0; i < n; i++)

if (!used[i])

{

dfs(i);

res++;

}

Print the answer.

printf("%d\n",res);

**Java implementation – dfs**

**import** java.util.\*;

**public** **class** Main

{

**static** **int** *g*[][], *used*[];

**static** **int** *n*;

**static** **void** dfs(**int** v)

{

*used*[v] = 1;

**for**(**int** i = 1; i <= *n*; i++)

**if** (*g*[v][i] == 1 && *used*[i] == 0) *dfs*(i);

}

**public** **static** **void** main(String[] args)

{

Scanner con = **new** Scanner(System.***in***);

*n* = con.nextInt();

*g* = **new** **int**[*n*+1][*n*+1];

*used* = **new** **int**[*n*+1];

**for**(**int** i = 1; i <= *n*; i++)

**for**(**int** j = 1; j <= *n*; j++)

*g*[i][j] = con.nextInt();;

**int** res = 0;

**for**(**int** i = 1; i <= *n*; i++)

**if** (*used*[i] == 0)

{

*dfs*(i);

res++;

}

System.***out***.println(res);

con.close();

}

}

**Java implementation – dsu**

**import** java.util.\*;

**public** **class** Main

{

**static** **int** *mas*[];

**static** **int** *n*;

**static** **int** Repr(**int** n)

{

**while** (n != *mas*[n]) n = *mas*[n];

**return** n;

}

**static** **void** Union(**int** x,**int** y)

{

**int** x1 = *Repr*(x);

**int** y1 = *Repr*(y);

**if** (x1 == y1) **return**;

*mas*[x1] = y1;

}

**public** **static** **void** main(String[] args)

{

Scanner con = **new** Scanner(System.***in***);

*n* = con.nextInt();

*mas* = **new** **int**[*n*+1];

**for**(**int** i = 1; i <= *n*; i++) *mas*[i] = i;

**for**(**int** i = 1; i <= *n*; i++)

**for**(**int** j = 1; j <= *n*; j++)

{

**int** val = con.nextInt();

**if** (i > j) **continue**;

**if** (val == 1) *Union*(i,j);

}

**int** count = 0;

**for**(**int** i = 1; i <= *n*; i++)

**if** (*mas*[i] == i) count++;

System.***out***.println(count);

con.close();

}

}

**Python implementation – dsu**

The function ***Repr*** returns the representative vertex of the set to which vertex *n* belongs. To do this, we follow the pointers sequentially until we reach the representative of the set (a vertex whose pointer points to itself).

**def** Repr(n):

**while** (n != mas[n]): n = mas[n]

**return** n

The ***Union*** function merges two sets containing vertices *x* and *y*. First, their representatives are found – let them be *x*1 and *y*1. If *x*1 = *y*1, then the vertices already belong to the same set, and no further action is required. Otherwise, the pointer of the representative *x*1 is redirected to *y*1.

**def** Union(x, y):

x1 = Repr(x)

y1 = Repr(y)

**if** (x1 == y1): **return**

mas[x1] = y1

The main part of the program. Each vertex initially points to itself (mas[*i*] = *i*).

n = int(input())

mas = [int(i) **for** i **in** range (n + 1)]

Read the adjacency matrix. For each edge (*i*, *j*) where *i* < *j*, perform a union of the sets containing vertices *i* and *j*.

**for** i **in** range(1,n + 1):

lst = [0] + [int(j) **for** j **in** input().split()]

**for** j **in** range(1, n + 1):

**if** (i < j **and** lst[j] == 1): Union(i, j)

The variable *res* stores the number of connected components. This number is equal to the number of representative vertices of the sets – that is, those vertices that point to themselves.

res = 0

**for** i **in** range(1, n + 1):

**if** (mas[i] == i): res += 1

Print the answer.

print(res)

**Python implementation – dfs**

The ***dfs*** function performs a depth-first search of the graph, starting from vertex *v*.

**def** dfs(v):

used[v] = 1

**for** i **in** range(n):

**if** g[v][i] **and not** used[i]:

dfs(i)

The main part of the program. Read the input data.

n = int(input())

g = [[0] \* n **for** \_ **in** range(n)]

used = [0] \* n

**for** i **in** range(n):

row = list(map(int, input().split()))

**for** j **in** range(n):

g[i][j] = row[j]

The variable *res* stores the number of connected components.

res = 0

A depth-first search is performed on the disconnected graph. Each call to the ***dfs*** function starts from an unvisited vertex, so the number of ***dfs*** calls equals the number of connected components in the graph.

**for** i **in** range(n):

**if not** used[i]:

dfs(i)

res += 1

Print the answer.

print(res)